THE 15th CHESAPEAKE SAILING YACHT SYMPOSIUM

Schooner *Brilliant* Sail Coefficients and Speed Polars

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ABSTRACT

The sail coefficients for a schooner rig, as a function of wind angle and heel angle, are presented, based on an experimental program, for historic vessel research, at Mystic Seaport, using the 61'6" schooner *Brilliant*. The coefficients were determined by full-scale sailing tests and $\frac{1}{9}$- scale model tow-tank tests.

Sail coefficients C_R and C_H are defined as the drive force and horizontal side force, due to the sails, rigging, and hull above the waterline, per unit of sail area, per unit of wind pressure. These coefficients can be used to study performance of historic schooner-rigged vessels, predict performance of new designs, and compare performance of schooners and sloops. Sail coefficients for sloops have long been available.

A velocity prediction program for the schooner was also developed. The predicted and actual ship speeds agree, with standard deviation of 0.028 in the ratio.

Upwind sail coefficients for the schooner are found to be lower than for historic sloops, and display the expected droop with heel. The schooner velocity made good upwind is largest with the sail plan of four lowers plus fisherman staysail. The schooner and sloop both point higher as wind increases. The sloop outpoints the schooner at all wind speeds, by about 10°.

On a beam reach or broad reach, schooner speed is largest with the sail plan of big jib, golliwobbler, and mainsail. This sail plan also produces the largest downwind velocity made good. The polars suggest that the schooner has the advantage over the sloop on a beam reach.

NOTATION

a	Height above DWL to point of model attachment to towing carriage	ft
a1	Constant in $CR^*=(a1+b1\ \beta)(1-.01446\ \phi)$	---
A	Sail area, actual	ft^2
A*	Fixed reference sail area. For *Brilliant*, 4 lowers & uncovered fisherman = 2469	ft^2
b	Height above DWL to metacenter	ft
b1	Constant in $CR^*=(a1+b1\ \beta)(1-.014456\ \phi)$	deg^{-1}
c	Exponent of ϕ in leeway equation	---
c1	Function of β in $CH^*= c1(1-.01795\ \phi)$	---
C_H	Sail side-force coef $= H_q/(0.5\rho_{air}U_a^2A)$	---
C_H*	Sail side-force coef, ref. sail area = $C_H(A/A^*)$	
C_R	Sail drive-force coef = $R_q/(0.5\rho_{air}U_a^2A)$	---
C_R*	Sail drive-force coef, ref. sail area = $C_R(A/A^*)$	
CSYS	Chesapeake Sailing Yacht Symposium	---
C_t	Coef of total hull resistance (drag)	---
CAH	Height above DWL to sail center of area	ft
CEH	Height above DWL to sail center of effort	ft
d	Zero intercept, side force equ, $F_e = d + nM_e$	lbf
D	Rudder angle (+ = weather helm)	deg
DL	Davidson Lab, Stevens Inst of Tech	---
DWL	Design waterline	---
e	Base of natural logarithms = 2.71828	---
f	Function of (N_{Fr}, ϕ) in leeway equation	deg
F	Horizontal side force relative to heading	lbf
g	Gravitational constant = 32.16	ft/s^2
g_m	Depth below model DWL to c.g. (g_m is negative since the *Brilliant* model c.g. is above the DWL), = -.1192	ft
g_s	Depth below ship DWL to the c.g.,= 1.034	ft
H	Horizontal side force relative to path	lbf
I	Righting Moment	ft lbf

k	Form factor in hull friction coef. equation	---
L	Design waterline length	ft
L_s	Hull sailing waterline length = 50.43	ft
L_1	Coef of leeway in attitude drag equation	$(deg)^{-1}$
LRD	Depth below DWL to center lateral resis.	ft
M	Transverse moment, Equ. (A17),(A18)	ft lbf
MSM	Mystic Seaport Museum	---
n	Slope in heel force equation: $F_e = d + nM_e$	ft^{-1}
N_{Re}	Hull Reynolds No. = $0.7U_sL_s/\nu$	---
N_{Fr}	Hull Froude No.=$U_s/(L_sg)^{0.5}$=$.04191V_s$	---
P_2	Coef of heel in attitude drag equation	$(deg)^{-1}$
R	Hull resistance force in path	lbf
S	Hull wetted surface area; 904	ft^2
SWL	Sailing waterline	---
U_a, V_a	Appar. wind speed (U_a=1.6878V_a)	ft/s, knots
U_s, V_s	Ship speed	ft/s, knots
U_t, V_t	True wind speed	ft/s, knots
VMG	Velocity made good	knots
VPP	Velocity Prediction Program	---
W	Sliding weight on model	lbf
X	Rate of change of λ with F_e	deg/lbf
Y	Distance that weight W is moved laterally	ft
β	BETA, apparent wind angle rel. to heading	deg
β_h	Reading of wind angle indicator at foremast head, uncorr. for heel and upwash	deg
Δ	DELTA, displ. (hull, SWL) = 94242	lbf
γ	GAMMA, true wind angle, relative to path	deg
ϕ	PHI, heel angle	deg
λ	LAMBDA, leeway angle (LWY in VPP)	deg
ν	NU, Kinematic viscosity	
	Fresh water ν_m = 1.0800(10^{-5}) at 68°F	ft^2/s
	Salt water ν_s = 1.1373(10^{-5}) at 68°F	ft^2/s
ρ	RHO, Fluid Density:	
	Air ρ_{air}=0.002335 at 68°F	$lbf\ s^2/ft^4$
	Fresh water $\rho_{w,m}$=1.937 at 68°	$lbf\ s^2/ft^4$
	Salt water $\rho_{w,s}$ = 1.988 at 68°F	$lbf\ s^2/ft^4$

Subscripts, Unless Otherwise Indicated Above

$( )_a$	due to hull attitude (λ and ϕ)
$( )_m$	model
$( )_o$	at rest, ship or model
$( )_q$	at sailing equilibrium of forces and moments
$( )_{ref}$	value when D =0 and CEH = 27.75 ft.
$( )_s$	ship
$( )_e$	value expanded from model to full-scale ship
$( )_r$	due to rudder angle
$( )_u$	upright ship condition
$( )_v$	due to friction
$( )_w$	due to upright wave-making
$( )_{60}$	at foremast head, 60 ft above DWL

Notes

$\delta( )$ indicates a small change in the quantity.
Multiple subscripts are separated by commas.
Sin, Cos, Sec are trigonometric functions.
Ratio, salt to fresh water density taken = 1.0263.
Scale factor, ship to model, is 9.00.
To expand inertial forces (buoyancy, wave-making, lift, drag due to lift) from model to ship at same N_{Fr}, multiply by 1.0263 $(9)^3$.
To expand moments of inertial forces from model to ship at same N_{Fr}, multiply by 1.0263 $(9)^4$.
All sail coefficients are based on the apparent wind at height of upright CEH.
All wind speeds and wind angles are taken at the CEH unless otherwise specified.

INTRODUCTION

Brilliant (Figure 1 and Figure 2) was designed by Olin Stephens and built by Nevins in 1931. She is a fast and seaworthy schooner design of the period and is still in active service as a sail training ship at MSM. For example she won the August, 2000, tall ships translatlantic race from Halifax Nova Scotia, to Cowes, England.

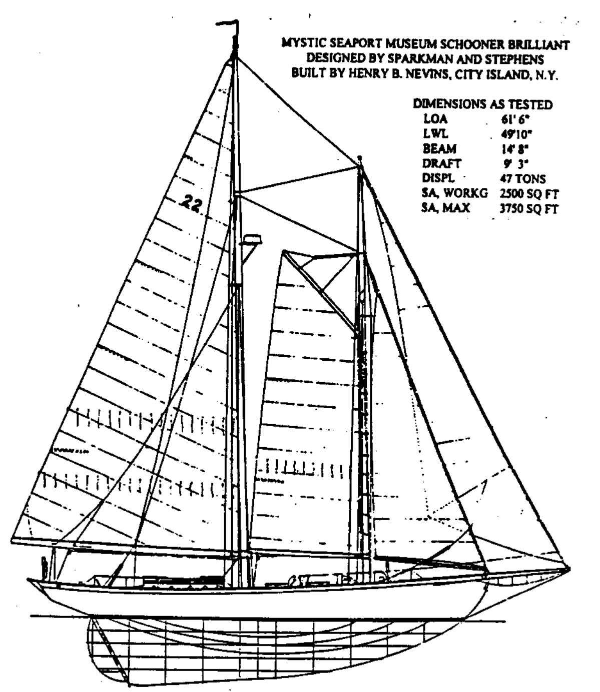


Figure 1.

Sail drive force coefficients and sail horizontal side force coefficients, a velocity prediction program (VPP). and speed polars are presented for Schooner *Brilliant*, as determined in a Mystic Seaport project for historic vessel research. Schooner sail coefficients, not previously available, were determined by full-scale sailing tests and $\frac{1}{9}$- scale model tow-tank tests.

The mathematical definitions of sail coefficients are:

$C_R = R_q/(.5\rho_{air}U_a^2A)$= Sail Drive Force Coef, actual(1)
$C_R{*}=R_q/(.5\rho_{air}U_a^2A{*})$= Sail Drive Force Coef, ref. (2)
$C_H = H_q/(.5\rho_{air}U_a^2A)$= Sail Side Force Coef, actual (3)
$C_H{*}=H_q/(.5\rho_{air}U_a^2A\ )$= Sail Side Force Coef, ref. (4)

The wind dynamic pressure, $0.5\rho_{air}U_a^2$ here is defined using the apparent wind speed U_a at the CEH (height, above the design waterline, DWL, of the center of sail effort), following the practice in the sloop programs in the literature: *Gimcrack* (Davidson, SNAME, 1936), *Baybea* (Kerwin, Oppenheim, Mays, MIT Pratt Project, 1974) and *Standfast* (Gerritsma, Kerwin, Moyes, MIT AYRU, 1975).

Figure 3 identifies the sail code numbers and three of the many sail plans tested. Table 1 lists sailing ranges of apparent wind angle over which each sail plan was tested, sail areas, and CEH values. CEH was taken to be 1.03 times the CAH (height, above the DWL, of the geometric center of sail area) to account to some extent for the increase in wind speed with altitude.

Figure 4 is the velocity triangle diagram and the force diagram in the horizontal plane at the CEH.

The test method deduced drive force, side force, and leeway angle for each condition of ship speed, heel angle, and rudder angle by expanding the tow tank forces. The sloop programs used the same method

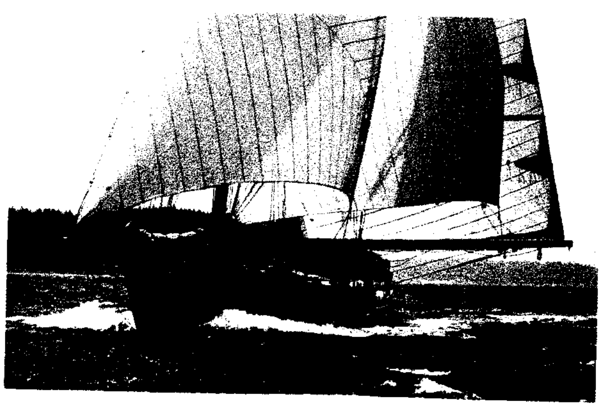


Figure 2. *Brilliant*
(Photo by Benjamin Mendlowitz, cover of *Wooden Boat* calendar.)

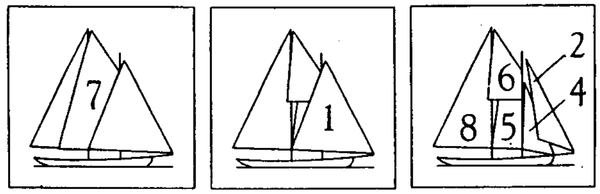


Figure 3. Sail code numbers

Table 1
Sail Plan Particulars

SAIL CODES

1 No. 1 (balloon) jib	5 Foresail
2 Jib	6 Fisherman staysail
3 Small jib (not tested)	7 Golliwobbler
4 Fore staysail	8 Mainsail

Suffix R indicates reefed sail

SAIL PLAN AREA AND CEH

Sail Plan	A, ft^2	CEH, ft
2458	2156	27.75
24568	2469	31.01
24568R	2253	30.17
2458R	1940	26.41
178	3167	30.40
1568	2628	30.45
24578	3429	30.21
258	1993	27.68
1578	3588	29.84

SAILING RANGES OF β AND γ

Sail Plan	β deg	β_h approx. deg	γ at $V_{t,60}$ = 5 kts deg
2458	20- 68	29 - 75	40-123
24568	28-107	36 -112	58-149
178	68-140	75 -144	136-146
1568	36- 65	45 - 73	65-117
24578	75- 94	83 -101	129- 41

Sail Plan	γ at $V_{t,60}$ = 7.5 kts deg.	10 knots deg	15 knots deg
2458	35 -120	35 -120	35 -107
24568	55 -149	53 -149	50 -143
178	133-160	127-160	111-159
1568	63 -114	62-111	65 -103
24578	126-140	124-139	114 -133

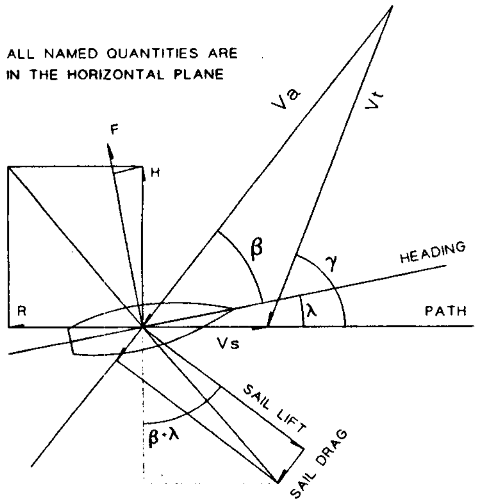


Figure 4. Velocity triangle and horizontal forces.

FULL SCALE SAILING TESTS

The sailing tests were described in the previous CSYS paper "On Test Measurements In Full Scale Sailing Test Programs" (Grant and Stephens, 1997). Sailing runs were carried out in smooth water, principally on Long Island Sound near Mystic, CT.

CALCULATION OF APPARENT WIND AT CEH

The true wind speed and angle were first determined in the horizontal plane at the anemometer location at the 60' foremast head height, by solving the wind triangle consisting of measured apparent wind (corrected for heel and upwash), measured boat speed, and a leeway angle calculated from the tank tests. Attempts to measure leeway directly during full-scale sailing tests were not satisfactory. This wind speed was then scaled, by the 0.1 power of height, (J. Beam, 1997) to the CEH. The true wind angle was assumed to be independent of height. The wind triangle at the CEH was then solved backward. Apparent wind speed was 5% to 15% lower and apparent angle a degree or two lower than at 60'.

MODEL TESTS IN THE TOW TANK

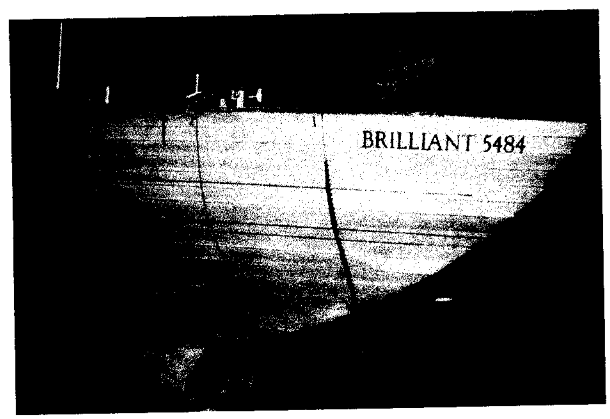
About 100 runs were conducted, including many repeats, using a 1/9 scale model in fresh water in Tank 3 at DL (Davidson laboratory, Stevens Institute of Technology), over the following ranges:

Speed: 1 to 3.5 knots (equivalent to full scale ship speed of 3 to 10.50 knots for same N_{Fr})
Leeway Angle: 0 to 8°
Heel Angle: 0 to 30°
Rudder Angle: 0 to 8°

The underwater lines of the ship, measured by International Measurement System in 1991, agreed closely with the 1931 table of offsets. The model (Figure 5) for tank tests was constructed at MSM to the 1931 offsets. Lines were measured at MSM in 1998 on the model and the ship, using a mechanical device (Saro Digital Arm) for the model and an optical transit (Sokkia Model 3000) for the ship. These measurements confirmed the model accuracy. Hull displacements agreed within about 1%.

To check for water absorption the model was weighed from time to time during tank testing. There was no measurable change in weight throughout the program.

Turbulence stimulating strips were applied to the model forebody (Figure 5). The strips are layers of black tape serrated at the leading edge to generate turbulence streaks that spread and join in the boundary layer over most of the underbody. The purpose was to approximate the boundary layer on the full scale ship hull, where transition from laminar to turbulent occurs spontaneously on the forebody due to the higher N_{Re}.



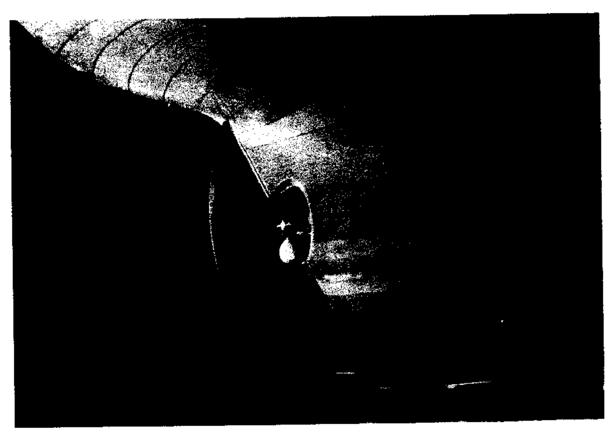


Figure 5. Model, showing turbulence stimulators,

The model was fitted with a movable rudder, propeller aperture, and one of two propeller blades. Previous testing at DL had indicated that modelling two blades overstated the propeller drag.

The model was immersed to the sailing water line. The fixed quantities in each run were speed, leeway angle, heel moment, and rudder angle. Quantities ed measured of resistance force, R, in the path, horizontal side force, H, relative to path, moments in the transverse plane and longitudinal plane, longitudinal trim angle, heel angle, and the vertical heave.

The model was allowed to rotate freely in heel and longitudinal planes, and to rise or fall. Bow-down trim was achieved by moving a weight forward, simulating the longitudinal rig moment. Heeling moment was set by moving a weight transversely. For each heel angle the tow-bar was reset to keep it on path axis.

CALCULATION OF C_R

To determine the sail drive coefficient C_R, it is only necessary to expand the hull resistance measured in the tank, at the correct condition of speed, heel, leeway, and rudder angle, to full scale ship resistance and insert this value in Equation (1) The method used here is the simplest of the several methods described by Teeters, CSYS, 1993.

There are two contributions to hull resistance: friction and inertial forces. Frictional resistance was assumed to scale with N_{Re} . Wave-making and other inertial forces were assumed to scale with N_{Fr}. N_{Re} was based on 0.7 L_s, a reasonable characteristic underwater length. N_{Fr} was based on L_s.

The frictional resistance is taken to be the product of flat plate drag coefficient, calculated from the Schoenherr Equation (Appendix 1), increased slightly by a form factor that accounts for hull curvature, multiplied by the the actual wetted surface area of the hull. In the case of the model, there is another small increment due to the turbulence stimulators.

The resistance due to wave-making and other inertial forces is taken as the sum of a value for the upright ship, plus a large increment due to leeway angle, a small increment due to heel angle, and a small increment due to rudder angle. The combination of leeway and heel is here referred to as attitude.
At very low speeds (ship or model) the wave-making resistance is assumed to be zero. The total upright resistance at very low speeds can then be assumed equal to the frictional resistance.

Features of *Brilliant* resistance are shown in Figures 6 and 7. Figure 6 shows the data and derived curve of model upright resistance , versus N_{Fr}. The fall-off in the data at N_{Fr}. below 0.14 is attributed to failure of the turbulence stimulation. At higher values of N_{Fr} the fit is good. This curve is also in Figure 7, along with the expanded upright resistance for the ship, and sample points with leeway, heel, and rudder added in.

Mathematical details are in Appendix 1 along with empirical equations for the inertial resistance components due to upright wave-making, leeway, heel, and rudder, deduced from the tank tests.

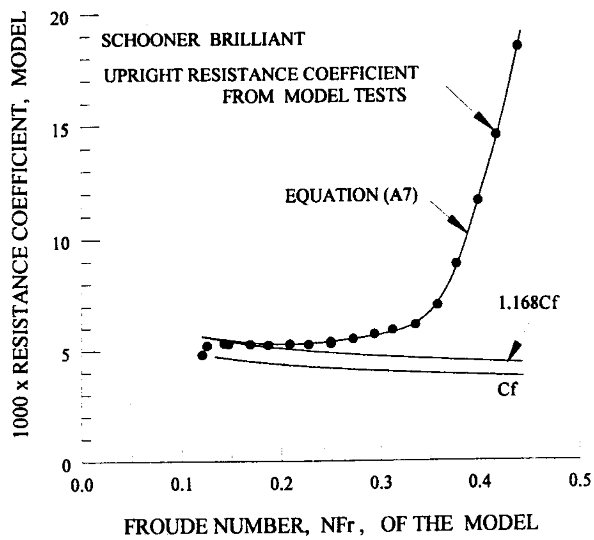


Figure 6. Equation (A7) is a good fit to the measured upright resistance coefficient data for the model.

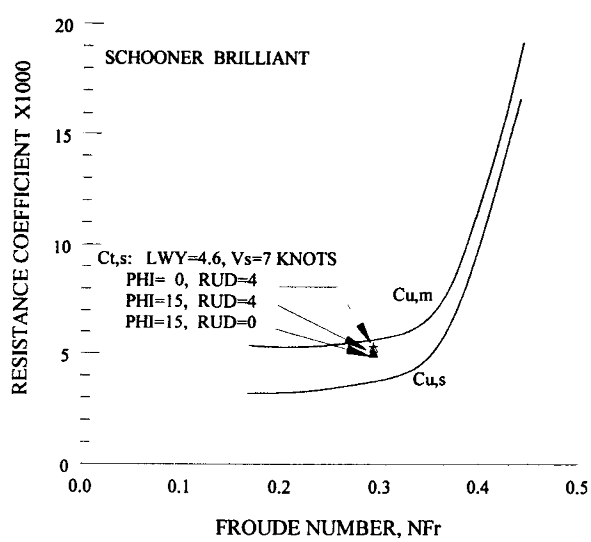


Figure 7. The effect of leeway on total resistance is large. The effects of heel and rudder are small.

CALCULATION OF C_H, λ_q, AND LRD

To calculate C_H requires determining H_q, the side force at sailing equilibrium, relative to path, and inserting this value in Equation (2). The side force F_q relative to heading was calculated first, since the heeling and righting moments are then more easily visualized. It will be shown that once F_q is known, then leeway λ_q can be determined, and H_q can be then be calculated:

$$H_q = (F_q - R_q \text{Sin}\lambda_q)/ (\text{Cos}\ \lambda_q) \qquad (5)$$

Calculation of $F_{q,ref}$

Figure 8 is the usual diagram of side forces and moment arms. The c.g. height is known for model and ship, as well as the CEH of the sails. The center of buoyancy is the same, to scale, for the model and ship.

The equations for forces and moments are in Appendix 2. Equilibrium is visualized by plotting side force F versus a transverse moment M. This approach was described in Davidson, 1937. Since LRD and b_m are unknown initially, and since g_e and g_s are not equal, then M is defined as the adjusted moment that would be found if g_e=g_s=0. For the expanded model data, M=M_e= the hull adjusted righting moment. For the ship, M=M_s= the rig adjusted heeling moment. M_e must equal M_s for sailing equilibrium.

Figure 9 is an example of the F-M diagram, for *Brilliant,* for ship speed of 7 knots at an arbitrary reference condition of zero rudder angle and 27.75' CEH. The hull characteristics expanded from tank data, F_e versus M_e, are the lines with negative slope. These hull characteristics are shown for λ=0, 2, 4, 6, 8°, and ϕ=0, 5, 10, 15, 20, 25°. The rig characteristics are straight lines with positive slope. Examples are shown for ϕ= 0, 5, 10, 15, 20, 25°. The intersection of a rig characteristic line and a hull characteristic line at the same heel angle is a point of sailing equilibrium..

These intersections in Figure 9 are denoted by ($F_{q,ref}$, $M_{q,ref}$). A curve drawn through all these intersection points is the sailing equibrium line for 7 knots. This line is shown in Figure 9 and by itself in Figure 10, for clarity. Also shown in Figure 10 are the ($F_{q,ref}$, $M_{q,ref}$) points expanded from tank data at V_s of 5 knots and 9 knots. These coincide with the 7 knot curve, as expected, providing a check on the method.

Algorithms for $F_{q,ref}$ and $H_{q,ref}$ versus λ and ϕ are developed in Appendix 2, based on F-M diagrams.

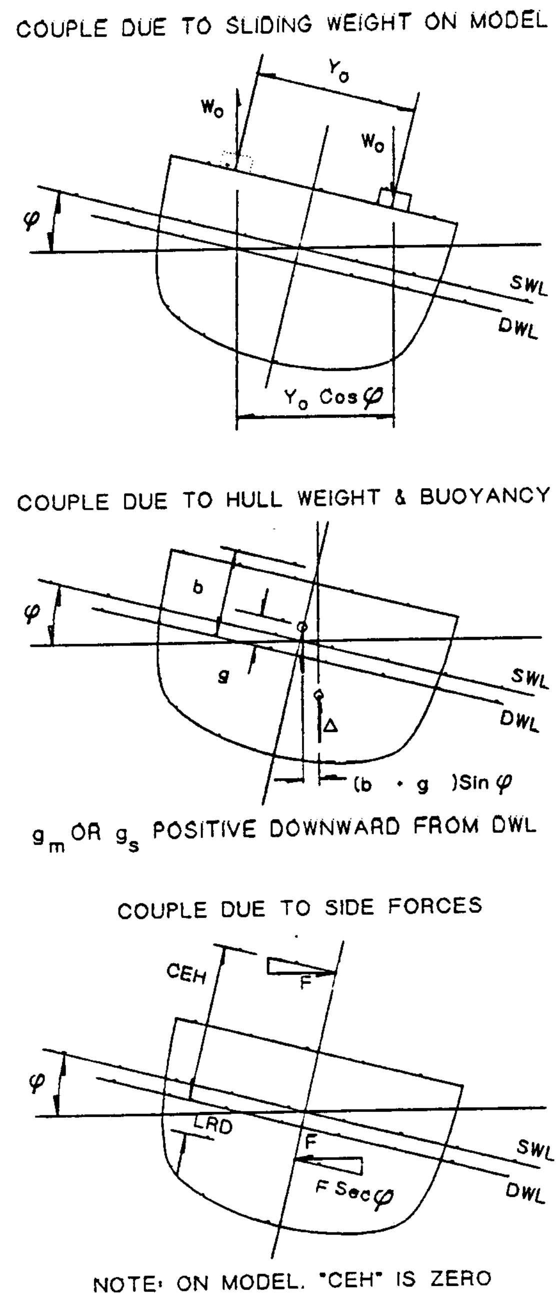


Figure 8. Transverse force and moment diagram

Calculation Of $\lambda_{q,ref}$

The sailing equilibrium value of leeway, $\lambda_{q,ref}$, is determined by $F_{q,ref}$, for a given combination of ϕ and V_s, by interpolation along a line of hull ϕ=constant in an F-M plot. For *Brilliant* an empirical equation for $\lambda_{q,ref}$ versus ϕ and V_s has been developed from the data, Equation (A19) in Appendix 2. Leeway is found to be approximately proportional to (heel)/($V_s^{1.8}$).

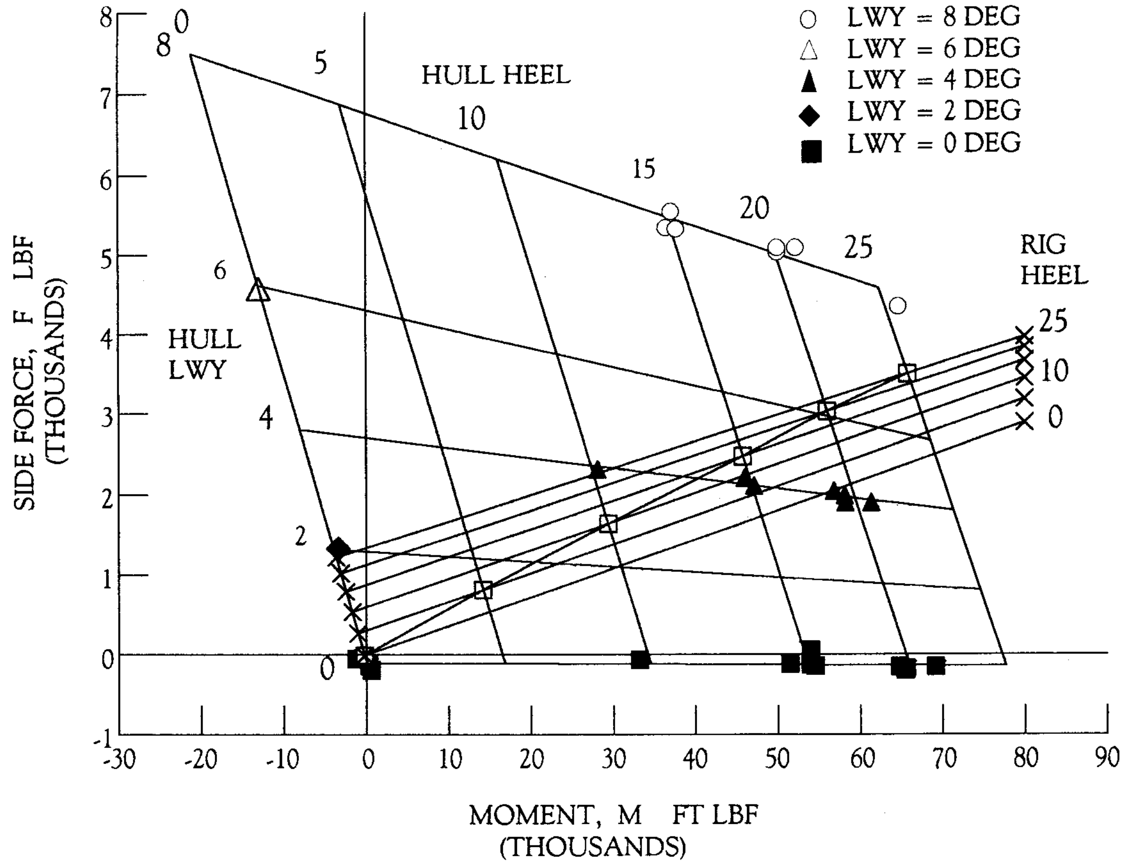


Figure 9. The F-M diagram at V_s=7 knots, 0 rudder, 27.75' CEH, establishes the reference sailing equilibrium line (open square symbols).

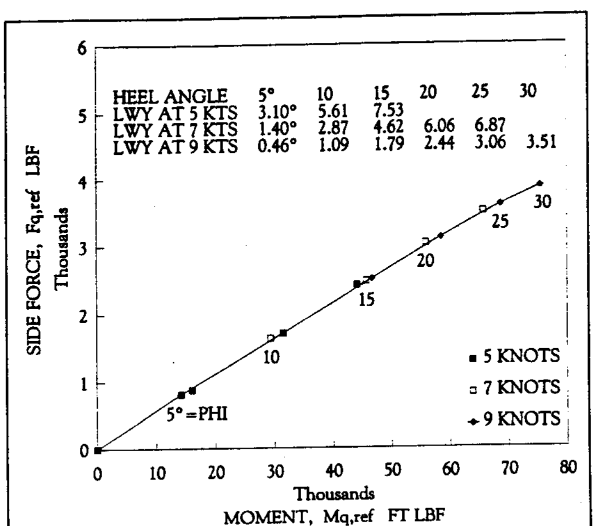


Figure 10. The reference sailing equibrium line defines leeway as a function of ship speed and heel angle.

Final Values for F_q and λ_q

Final F_q is obtained by adding to $F_{q,ref}$ an increment due to any CEH change. Final λ_q is obtained by adding to $\lambda_{q,ref}$ an increment due to rudder and an increment due to CEH change. The empirical equations are given in Appendix 2.

Final Calculation of C_H

Now that final F_q and λ_q are known, H_q can be then be obtained from (5) and inserted in (3) to obtain C_H.

CALCULATION OF LRD

In Appendix 3 it is shown that the slope of the hull characteristic line F_e versus M_e is a measure of LRD. It was found that the slope varies slightly with ship speed and heel angle. The empirical equation (A35) in Appendix 3 permits calculation of LRD for known ship speed and heel angle. The resulting values are in the vicinity of 32% of *Brilliant* maximum draft.

STATIC RIGHTING MOMENT

As a check on hull form and calculation procedures, static righting moment for model and ship were calculated and compared. When the model is at rest in the tank, immersed to the SWL , and is heeled using a sliding weight, F_m=0 and the moment equation is

$$YW(\cos \phi_o) = g_m(\sin \phi_o)\Delta_m + b_m(\sin \phi_o)\Delta_m \quad (6)$$

The subscript $( )_o$ always refers to the at-rest condition. The right side of (6) is the model righting moment. For the full-scale ship, immersed to SWL, heeled, the expanded equation for righting moment, $I_{o,e}$, is then

$$I_{o,e} = g_s(\sin \phi_o)\Delta_e + b_e(\sin \phi_o)\Delta_e \quad (7)$$

Subtract Equation (6) from (7), use b_s=b_e, Δ_s=Δ_e, and the known values of Δ_s, g_s and g_e, The result is

$$I_{o,e} = 6731.6\ [YW(\cos \phi_o) + 29.495\ (\sin \phi_o)] \quad (8)$$

Figure 11 compares this result with the righting moment curve determined from the ship's lines, using the Nautilus computer program:

$$I_{o,s} = [6064(\cos \phi_o) - 775][\phi_o] \quad (9)$$

The agreement between the computer prediction and the measured expanded model data in Figure 11 is excellent, with differences generally less then 1%.

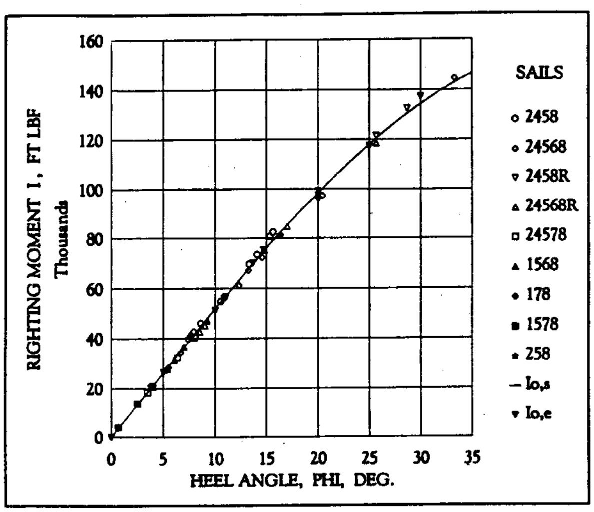


Figure 11. Static righting moment from expanded model data ($I_{o,e}$ symbols) and from ship lines calculations ($I_{o,s}$ line) agree closely. Both agree well with dynamic righting moment for 46 full-scale sailing test runs (symbols).

DYNAMIC RIGHTING MOMENT

The righting moment under sail, called dynamic righting moment, I_s, was compared with the static moment, I_o. I_s could in principle change with speed, leeway, heel, and rudder angle, because waterline shape is altered. Since LRD is known from (A35), I_s can be calculated directly for each sailing test point:

$$I_s = (CEH + LRD)(\sec \phi)(F_q) \quad (10)$$

The resulting values of I_s for each of the sailing tests of schooner Brilliant are also shown in Figure 11, along with the static data, versus heel angle. The differences from the static righting moment are small. The average difference is 0.1%. The standard deviation is 2.1% of I_s, which is about the same as the uncertainty in the dynamic moment determination

.

C_R AND C_H RESULTS

The sail coefficients were calculated at all 46 sailing test points. The results are listed in Table 2.

As an example of results, C_R* and C_H*, at β = 30°, are plotted versus ϕ, Figure 12. The effect of ϕ on C_R* is -1.446% per degree, and on C_H* -1.795% per degree.

For comparison with historic sloop results upwind, the *Brilliant* coefficients C_R and C_H, based on actual sail area , are plotted versus ϕ, Figure 13, for β near 30° , along with the corresponding sloop curves for *Baybea* and *Gimcrack*. The sloop *Standfast* results (not shown) were close to the *Baybea* results. The

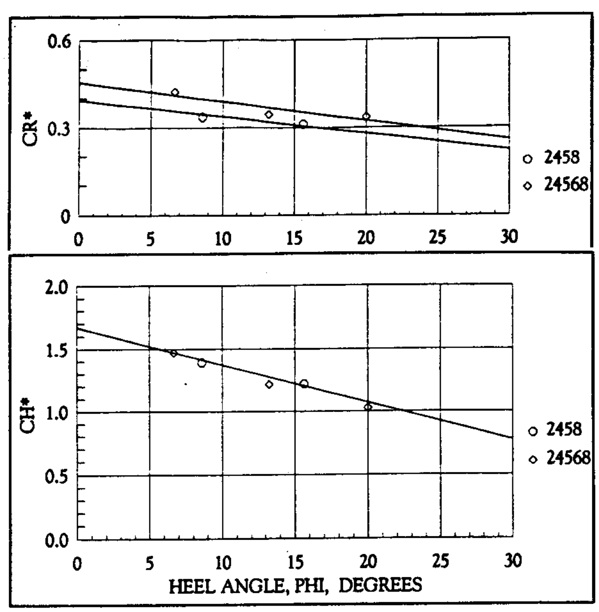


Figure 12. CR* and CH* decrease as heel increases

Table 2
Schooner Brilliant
Sail Coefficients For 46 Full-Scale Sailing Test Runs

Run No.	Sails	Tack and Day No.	Vs	ϕ	D Rud.	λ	γ	(Vt)60	Vt	ß	Va	VMG	Note	Sail Coefficients CR	CH	CR*	CH*
-	-		Knots	Deg	Deg	Deg	Deg	Knots	Knots	Deg	Knots	Knots		---	---	---	---
47	1 578	PORT6	5.113	0.68	1.9	0.10	165.36	10.68	9.96	150.77	5.173	-4.947	a	0.699	0.343	1.0163	0.4989
27	1 578	STBD5	5.969	2.54	3.5	0.57	172.38	10.29	9.59	159.62	3.763	-5.916	a	1.949	2.409	2.8320	3.5006
26	1 78	PORT5	9.881	16.33	7.9	1.00	109.59	15.12	14.12	67.86	14.269	-3.312		1.471	1.083	1.8868	1.3888
24	1 78	PORT5	7.936	n.meas	4.7		134.48	10.56	9.86	81.70	7.112	-5.561		1.349		1.7300	
25	1 78	PORT5	5.273	n.meas	3.3		148.35	9.04	8.45	113.37	4.830	-4.489		1.002		1.2851	
49	1 78	STBD6	5.310	4.00	3.7	1.40	157.09	11.55	10.80	136.37	6.256	-4.891		0.634	1.513	0.8135	1.9413
48	1 78	STBD6	5.338	3.80	4.7	1.25	159.45	11.29	10.55	139.54	5.861	-4.998		0.741	1.639	0.9500	2.1026
42	1 568	STBD6	3.746	6.99	0.6	4.55	65.55	6.33	5.92	36.47	8.211	1.550		0.310	1.827	0.3301	1.9446
44	1 568	STBD6	7.153	14.75	5.4	3.44	85.76	10.48	9.79	47.67	12.546	0.529	b	0.448	1.558	0.4772	1.6584
43	1 568	PORT6	6.467	9.21	3.7	2.75	87.57	9.05	8.46	48.32	10.864	0.274		0.436	1.355	0.4644	1.4419
46	1 568	PORT6	6.807	5.36	1.6	1.26	113.34	9.02	8.43	64.61	8.483	-2.697		0.719	1.319	0.7653	1.4041
45	1 568	STBD6	6.306	6.07	3.4	1.75	113.70	8.47	7.91	64.92	7.892	-2.534		0.720	1.720	0.7666	1.8313
5	24578	STBD3	7.615	6.33	4.8	1.07	126.06	11.03	10.29	78.34	8.467	-4.483		0.773	1.206	1.0729	1.6745
4	24578	STBD3	8.220	8.00	6.7	1.07	130.07	13.19	12.31	87.14	9.429	-5.291		0.845	1.218	1.1742	1.6921
4A	24578	STBD3	7.285	3.50	4.2	0.56	134.77	11.05	10.32	89.31	7.329	-5.131		0.895	0.899	1.2433	1.2489
34	24568	STBD5	6.305	13.24	6.0	3.98	54.47	9.66	9.04	28.50	13.703	3.664		0.325	1.236	0.3255	1.2361
7	24568	PORT3	4.955	6.67	4.7	2.91	61.80	5.89	5.51	29.82	8.989	2.342		0.400	1.513	0.3999	1.5133
8	24568	PORT3	7.907	20.00	8.1	3.12	61.19	13.25	12.41	34.93	17.636	3.811		0.315	1.048	0.3147	1.0480
1	24568	STBD1	7.805	33.30	10.9	4.04	65.73	16.79	15.72	41.08	20.221	3.208	(!)	0.246	1.014	0.2461	1.0140
6	24568	STBD3	5.273	5.50	3.4	2.19	72.23	8.31	7.78	41.91	10.651	1.609	(!)	0.291	0.893	0.2908	0.8930
6A	24568	STBD3	4.967	5.33	3.7	2.29	72.44	8.56	8.02	43.69	10.629	1.499	(!)	0.262	0.870	0.2623	0.8699
9	24568	PORT3	8.084	12.33	7.1	1.83	81.11	12.43	11.63	47.48	15.158	1.249		0.432	0.949	0.4320	0.9495
3A	24568	STBD2	8.571	14.56	8.8	1.70	94.33	14.52	13.59	59.18	15.510	-0.646		0.561	1.049	0.5611	1.0491
3	24568	STBD2	9.286	20.44	8.6	1.69	100.49	16.82	15.75	65.79	16.762	-1.691		0.856	1.171	0.8561	1.1713
15	24568R	STBD4	9.519	25.67	3.4	2.01	104.74	20.79	19.41	74.28	19.326	-2.421		0.825	1.148	0.7527	1.0474
11	24568R	STBD4	9.691	17.00	5.1	1.19	114.97	20.36	19.00	83.28	17.310	-4.091		1.200	1.079	1.0951	0.9847
12	24568R	STBD4	9.134	8.50	3.4	0.78	127.48	15.45	14.42	87.42	11.452	-5.558		1.695	1.328	1.5464	1.2120
13	24568R	STBD4	8.462	4.00	3.4	0.43	136.70	14.35	13.40	97.55	9.281	-6.158		1.491	0.974	1.3606	0.8887
14	24568R	STBD4	9.024	9.00	5.9	0.86	138.64	17.48	16.32	105.78	11.259	-6.773		1.620	1.452	1.4786	1.3251
32	245 8	PORT5	4.567	7.40	3.7	4.07	41.72	8.97	8.31	23.11	12.103	3.409		0.248	1.198	0.2162	1.0459
29	245 8	STBD5	4.574	7.92	3.2	4.44	45.86	8.09	7.49	24.33	11.168	3.185	c	0.305	1.501	0.2666	1.3108
33	245 8	PORT5	4.882	7.66	1.9	4.21	47.16	8.48	7.85	25.18	11.728	3.320		0.292	1.317	0.2553	1.1501
30	245 8	STBD5	5.065	8.60	3.6	4.36	54.86	8.08	7.48	28.78	11.195	2.915		0.365	1.614	0.3188	1.4094
50	245 8	STBD6	6.656	15.63	6.0	4.59	54.14	11.91	11.03	29.68	15.873	3.899		0.335	1.380	0.2929	1.2048
52	245 8	STBD6	6.910	15.30	5.2	4.23	59.80	11.92	11.03	33.20	15.690	3.477		0.351	1.387	0.3069	1.2114
51	245 8	PORT6	7.213	13.33	3.9	3.32	60.73	11.50	10.65	33.47	15.508	3.527		0.356	1.261	0.3104	1.1015
53	245 8	PORT6	7.540	14.10	3.9	3.09	63.89	12.17	11.27	35.90	16.080	3.319		0.362	1.233	0.3161	1.0767
31	245 8	PORT5	7.339	10.88	3.7	2.51	71.27	10.07	9.32	38.00	13.588	2.356		0.452	1.368	0.3950	1.1946
54	245 8	STBD6	7.149	10.50	4.0	2.59	71.47	10.94	10.13	40.22	14.135	2.272		0.395	1.223	0.3451	1.0680
28	2 5 8	STBD5	4.038	7.58	3.7	4.56	38.76	8.46	7.83	21.23	11.262	3.149	c	0.271	1.538	0.2186	1.2411
16	245 8R	PORT4	9.043	25.67	9.0	2.58	52.74	21.63	19.93	34.34	26.406	5.476		0.362	0.825	0.2842	0.6480
17	245 8R	STBD4	8.959	28.67	6.1	3.16	63.06	22.69	20.90	42.16	26.206	4.059		0.339	0.879	0.2666	0.6903
20	245 8R	PORT4	9.584	20.00	4.4	1.58	79.51	18.25	16.81	51.01	20.816	1.745		0.880	1.130	0.6916	0.8881
18	245 8R	PORT4	9.286	11.00	6.2	0.97	93.67	17.91	16.50	62.47	18.410	-0.594		0.889	0.874	0.6985	0.6867
19	245 8R	STBD4	9.516	14.67	5.1	1.18	102.49	18.43	16.98	69.39	17.578	-2.057		1.177	1.238	0.9250	0.9726
10	245 8R	STBD4	9.380	13.67	5.1	1.19	109.29	17.66	16.26	74.18	15.867	-3.098		1.287	1.430	1.0116	1.1236

Notes:
a Winged Foresail (Unstable)
b Sailed Low
c Pinched
(!) Vs reading believed low (see Fig. 17)

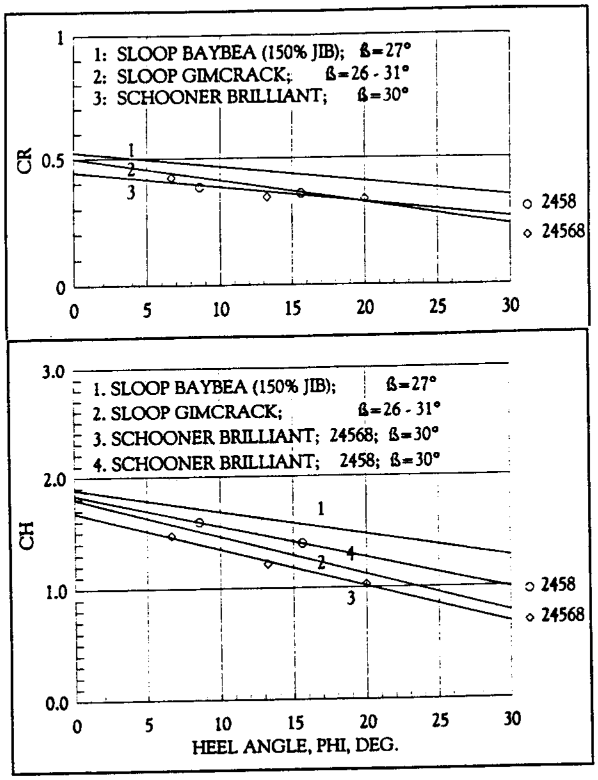


Figure 13. CR and CH versus heel angle for schooner and historic sloops follow similar trends.

conclusions are (1) the effect of heel is about the same for schooner and historic sloops, and (2) the upwind sail coefficients for schooner *Brilliant* are 5% to 10% lower than for sloop *Baybea,* and close to the sloop *Gimcrack* values. About one-twentieth of the droop with heel is believed to be due to the use of a fixed value of CEH, equal to the value for the upright ship.

Using the slopes versus ϕ from Figure 12, CR* and CH* at all 46 sailing test points were extrapolated to ϕ=0 and to ϕ=30°. Approximate trend lines are plotted in Figure 14, along with the actual sail coefficient values, versus β. Finally, curves of values for *Baybea*, calculated in the same way, are included in Figure 14. C_R* is smaller for schooner than sloop, except when reaching. (For beam reach, β is about 60°.) C_H* is smaller for schooner than sloop except broad reaching. The upwind differences in C_R* are partly due to the schooner's lack of a large efficient upwind headsail and to the greater windage of the rig. The differences reaching and downwind may be biased in favor of the schooner by the inclusion of of the golliwobbler.

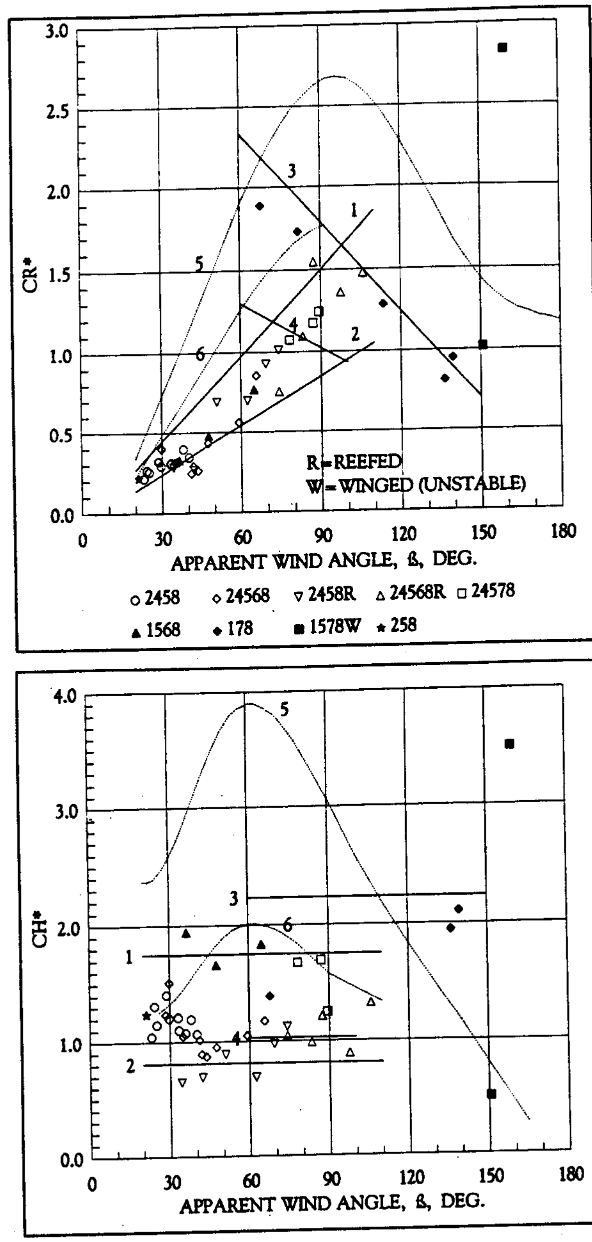


Figure 14. The trend lines indicate approximate values of CR* and CH*, obtained by extrapolating to heel angle ϕ of 0° and 30° as follows:

Curve 1. *Brilliant* sail plan 24568, ϕ = 0
Curve 2. *Brilliant* sail plan 24568, ϕ = 30
Curve 3. *Brilliant* sail plan 178, ϕ = 0
Curve 4. *Brilliant* sail plan 178, ϕ =30
Curve 5. Sloop *Baybea*, ϕ = 0
Curve 6. Sloop *Baybea*, ϕ = 30

Figure 14 provides only an approximate overview. To construct a VPP and a speed polar diagram it is necessary to develop detailed expressions for sail coefficients versus β and ϕ for each sail plan. As an example, Figures 15 and 16 show the data for sail plans 24568 and 178, versus β, and empirical curves.

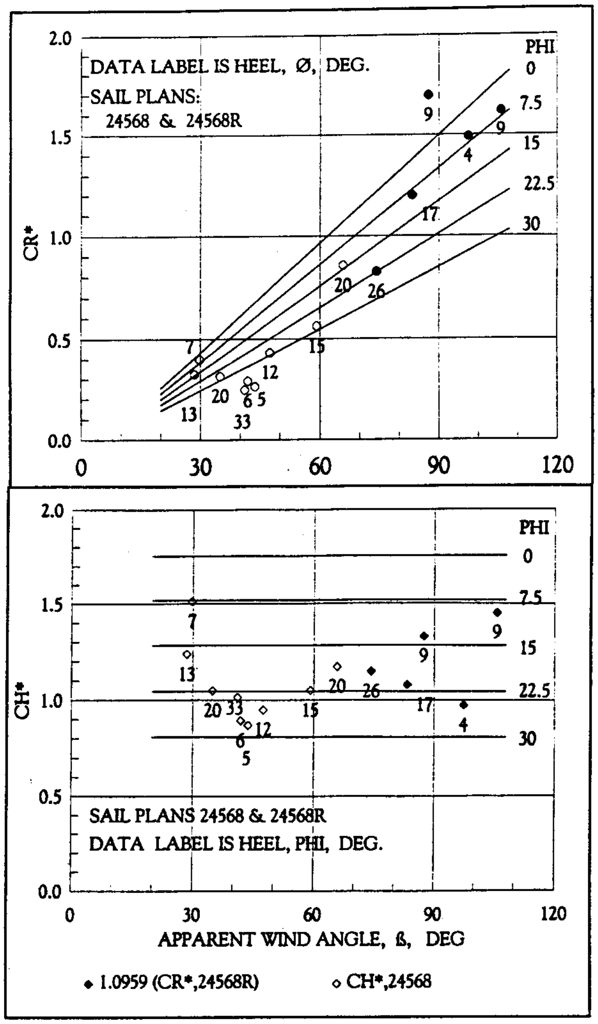


Figure 15. Sail plan 24568, CR* and CH* versus β

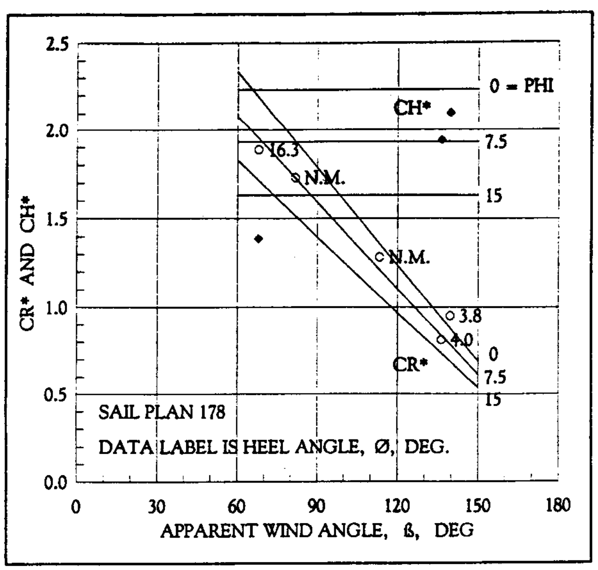


Figure 16. Sail plan 178, CR* and CH* versus β

The following discussion describes the procedure for developing the empirical equations for C_R* and C_H* for each sail plan, versus β and ϕ.

First, Figure 12 indicated that ϕ affected C_R* by the factor (1-.01446 ϕ) and C_H* by the factor (1-.01795ϕ).

Second, for each sail plan, at fixed heel angle, some function describing sail coefficient versus β was selected. Trials indicated that usually a straight-line variation with β over the range of interest was a reasonable fit, within the uncertainty of the data. An exception was C_H* for 2458 (four lowers only), where a negative β^2 term was added. A simplification was to assume that reefing the mainsail in sail plans 2458 and 24568 (resulting in 2458R and 24568R) simply scaled down the drive force and side force at all wind speeds and angles. Thus in Figure 15 the curves for 24568 and 24568R are the same when the sail coefficients for 24568R are multiplied by 1.0959.

Third, it was required that the predictions of ship speed, based on the use of these sail coefficient empirical equations in a VPP, must agree reasonably well with actual ship speeds at the full-scale sailing test points. The VPP is described in the next section. The resulting ratio of predicted to actual ship speeds is plotted versus true wind angle in Figure 17 for all the sailing tests. Standard deviation is a reasonable 0.028 after eliminating three outlying points taken early in the sailing program. This result tends to support the choice of the empirical equations for C_R* and C_H*.

Table 3 lists the final equations for C_R* and C_H*.

THE VELOCITY PREDICTION PROGRAM

Table 4 lists the VPP procedure developed for predicting the performance of schooner *Brilliant*, with all equations. The table also includes a sample calculation.

The basic approach is to choose $V_{t,60}$ and γ, guess V_s, ϕ, and C_f, then iterate V_s, ϕ, and C_f until the guessed values result in (a) hull resistance equal to sail drive force, (b) hull side force equal to sail side force and (c) the Schoenherr equation for C_f (see Appendix 1) is satisfied. Note that no iteration is performed on rudder angle, D. Instead D is assumed to depend on V_s and β:

$$D = -5.9 + 1.8V_s + (.052-.0105V_s)\,\beta \qquad (11)$$

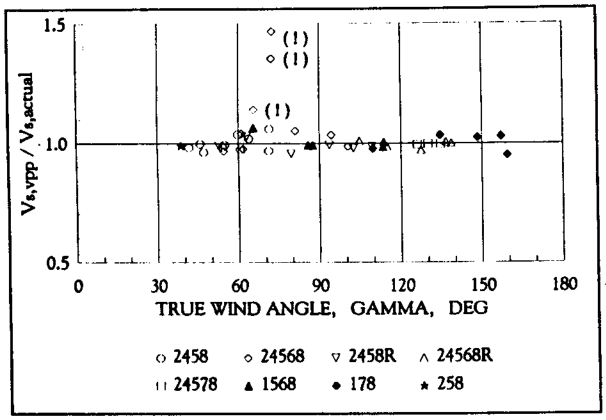


Figure 17. Predicted ship speed agrees with actual. The three points labeled (!) were outlying data points (see Table 2) that were discarded

Table 3

Empirical Equations For CR* and CH* Versus β and ϕ

Sails	Equation for CR*
2458	$(-.1480 + .0180\beta)(1 - .01446\phi)$
2458R	$CR^*_{2458} / 1.0526$
258	$CR^*_{2458} / 1.0818$
24568	$(-.1004 + .01776\beta)(1 - .01446\phi)$
24568R	$CR^*_{24568} / 1.0959$
178	$(3.429 - .01927\beta)(1 - .01446\phi)$
1568	$(-.2192 + .01643\beta)(1 - .01446\phi)$
24578	$(-.6300 + .01576\beta)(1 - .01446\phi)$

	Equation for CH*
2458	$(-.4937 + .10232\beta - .0008639\beta^2)(1 - .01795\phi)$
2458R	$CH^*_{2458} / 1.2500$
258	$CH^*_{2458} / .998$
24568	$1.7546 (1 - .01795\phi)$
24568R	$CH^*_{24568} / 1.0959$
178	$2.2330 (1 - .01795\phi)$
1568	$2.0300 (1 - .01795\phi)$
24578	$2.0300 (1 - .01795\phi)$

This equation was developed by examination of all rudder angle data from the sailing test program, and was found to accurate to about 1° in rudder angle.

The VPP procedure is as follows:

Enter $V_{t,60}$, γ, sail codes, CEH, and A* (2469 ft^2).

These quantities remain constant throughout the calculation of the performance point.

Enter initial guesses for V_s, ϕ, and C_f.
Iterate V_s until line 24 is zero ($R_{rig} = R_{hull}$).
Iterate ϕ until line 28 is zero ($H_{rig} = H_{hull}$).
Iterate C_f until line 11 = line 12 (transcendental equation for C_f satisfied).
Repeat all iterations until all three conditions are met simultaneously. Convergence is generally rapid, requiring about a dozen total iterations.

VPP RESULTS

The ship speeds predicted by the VPP calculations were examined versus true wind angle γ at four true wind speeds, $V_{t,60}$ = 5, 7.5, 10, and 15 knots, for several sail plans, and plotted as follows:

Figure 18. V_s versus γ, for 5 sail plans
Figure 19. VMG versus γ, for 3 sail plans
Figure 20. Speed polar diagram for the schooner.

Omitted are sail plans with reefed mainsail (2458R and 24568R), not used in this wind range, and two sail plans for which data was sketchy (258 and 1578).

In the polar diagram only the sail plans 24568 and 178 are presented, since 24568 results in the best speed and VMG upwind and 178 results in the best speed on reaches and best VMG downwind.

Finally, the published speed polar diagram for sloop *Baybea* is presented in Figure 21 for comparison.

For the sloop *Baybea* the working sails are mainsail and 100% jib. The *Baybea* authors mention that a big genoa jib improved performance only slightly at low wind speeds and not at all at high wind speeds.

Additionally, the sloop might be able to fly some combination of reaching spinnaker and fore staysail when reaching so no direct comparison of sloop and schooner is possible when light sails are set.

Table 4.
VELOCITY PREDICTION PROGRAM
FOR SCHOONER BRILLIANT

nu = 1.137(10^-5) ft²/s, except to calc (N)Re use CJH's 1.282
Lw = 50.43ft .5(rho)salt water = .994 lbf s²/ft^4
.5(rho)air = .001675 lbf s²/ft^4 Form Factor k = 1.168
S = 904 ft² (ft/sec) = 1.6878 (knots)

Line				
1	(Vt)60	kts	Input	15
2	γ	deg	Input=Gamma	35
3	D	deg	(-5.9+1.8Vs)+(.052-.0105Vs)ß	3.982
4	Sails	~	Input=sail codes	2458
5	Vs	kts	Input (Iterate until line 24 = 0)	5.543
6	ø	deg	Input (Iterate until line 28 = 0)	15.350
7	CEH	ft	Input	27.750
8	A*	ft²	Input=Standard Sail Area	2469
9	(N)Re	~	0.7(Us)(Lw)/(nu)=4647500 Vs	3E+07
10	Cf	~	Input (Iterate until line 11=12)	0.0025270
11	Left	~	.242/[(Cf)^.5]	4.814
12	Right	~	(Log)10 { [(N)Re][Cf] }	4.814
13	(N)Fr	~	Froude No.=(0.04191)(Vs)	0.2323
14	Cw	~	Subroutine lines 34-41	0.0003703
15	Cu	~	1.168Cf + Cw	0.0033218
16	λ	deg	Subroutine, lines 45-56	5.5525
17	L1	~	.035 - .0608 (N)Fr	0.020876
18	P2	~	-0.00116 (LWY)	-0.006441
19	Ca	~	{(L1)(LWY^2.1)+(P2)(ø)}{Cu}	0.0022093
20	Cr		((D/8)^1.35)(e^-(N)Fr/.132)(Ca+Cu)	0.0003711
21	Ct	~	Cu + Ca + Cr	0.0059022
22	R,hull	lbf	2560 (Vs)² (Ct)	464.24
23	R,rig	lbf	.0011675 (Ua²)(A*) (CR*,ceh)	464.28
24	(R,rig-R,hull)/R,rig			0.000
25	F		See line 54.	2494.6
26	H,hull	lbf	(F-R,h [Sin lwy])/Cos lwy	2461.3
27	H,rig	lbf	.0011675Ua²(A*)(CH*,ceh)	2460.4
28	(H,rig-H,hull)/H,rig			-0.000
29	Va		Subroutine: line 62	18.6997
30	ß		Subroutine: line 63	19.6585
31	CR*		See lines 42-43	0.1617
32	CH*		See line 44	0.8569
33	SUBROUTINE FOR Cw			
34			.0901217 [(N)Fn^4]	0.0002625
35			23.8328 [(N)Fn^8]	0.0002021
36			-4780.43 [(N)Fn^12]	-0.0001181
37			360437 [(N)Fn^16]	0.0000259
38			-1.0365 (10^7) [(N)Fn^20]	-0.0000022
39			1.02037 (10^8) [(N)F24^24]	0.0000001
40			1.22825 (10^7) [(N)Fn^28]	0.0000000
41	Cw		SUM	0.0003703
42	a1,b1 in CR*=(a+b ß)(1-.01446 ø)		a1	-0.14800
43			b1	0.01810
44	Factor c1 in CH* (c1)(1 - .01795 ø)		c1	1.18274
45	SUBROUTINE FOR F AND LWY			
46	c	~	1.6 {1-.037 ([|Vs - 5|]^1.41)}	1.5750
47	f	~	3.8 (ø^c)/(Vs^1.8)	12.8568
48	LWY01		f - .038 f² - .00028 f³	5.9804
49	Fq,1	lbf	(350 Cos ø - 175) ø	2494.62
50	dFd,1	lbf	31(D^1.22) + 0.613(D^1.447)(ø)	236.80
51	Xd,1	~	See Equ. (A25), appendix 2.	0.75392
52	dLWYd		-(Xd,1) (LWY,01) (dFd,1) /Fq,1	-0.42800
53	dF	lbf	-(Fq,1) (CEH-27.75)/27.75	0.000
54	F	lbf	Fq,1 + dF,ceh	2494.624
55	dLWYceh		Xd,1(LWY,01)((CEH/27.75)-1)	0.000
56	λ	deg	LWY,01+dLWYd,1+dLWYceh	5.5525
57	SUBROUTINE FOR Va AND ß			

50	dFd,1	lbf	31(D^1.22) + 0.613(D^1.447)(ø)	236.80
51	Xd,1	~	See Equ. (A25), appendix 2.	0.75392
52	dLWYd		-(Xd,1) (LWY,01) (dFd,1) /Fq,1	-0.42800
53	dF	lbf	-(Fq,1) (CEH-27.75)/27.75	0.000
54	F	lbf	Fq,1 + dF,ceh	2494.624
55	dLWYceh		Xd,1(LWY,01)((CEH/27.75)-1)	0.000
56	λ	deg	LWY,01+dLWYd,1+dLWYceh	5.5525
57	SUBROUTINE FOR Va AND ß			

58	γ - λ	deg	γ - λ	29.448
59	Vt	kts	(Vt)60 ((CEH/60^.1))	13.887
60	v	kts	Vt sin (gamma-lwy)-Vs sin lwy	6.290
61	w	kts	Vt cos(gamma-lwy)+Vs cos lwy	17.610
62	Va	kts	(v²+w²)^0.5	18.700
63	ß	deg	IF(w<0,180-aSin(v/Va),aSin(v/Va))	19.658
64			SUMMARY	
65	(Vt)60	kts	LINE 1	15
66	γ	deg	LINE 2	35
67	Sail Plan		LINE 4	2458
68	Vs	kts	LINE 5	5.543
69	ø	deg	LINE 6	15.35
70	ß	deg	LINE 30	19.658
71	VMG	kts	(LINE 5)(COS LINE 2)	4.541
72	λ	deg	LINE 16	5.552

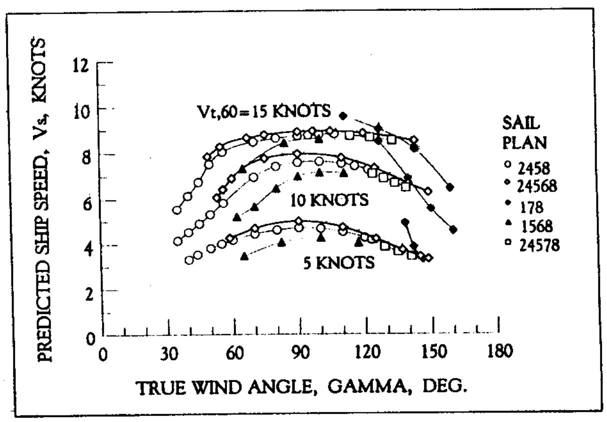


Figure 18. Sailplan for best V_s is 24568 upwind, 178 downwind.

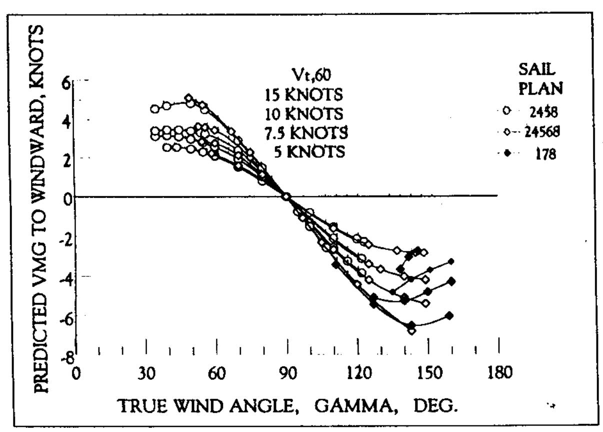
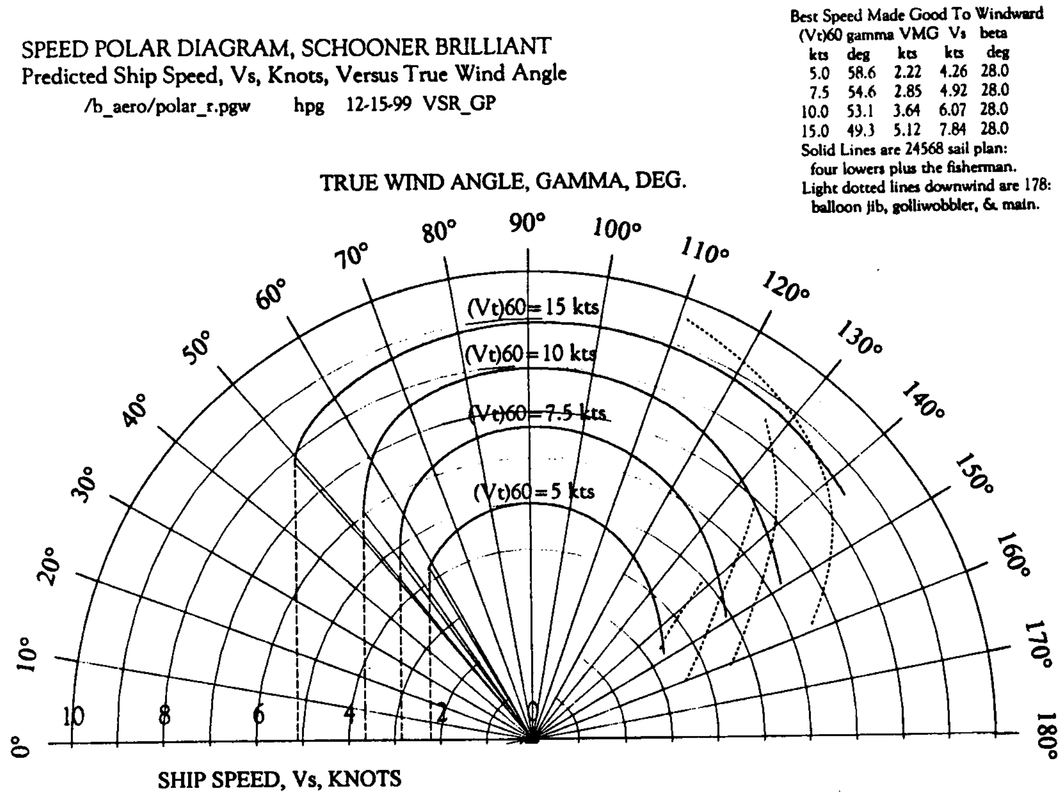
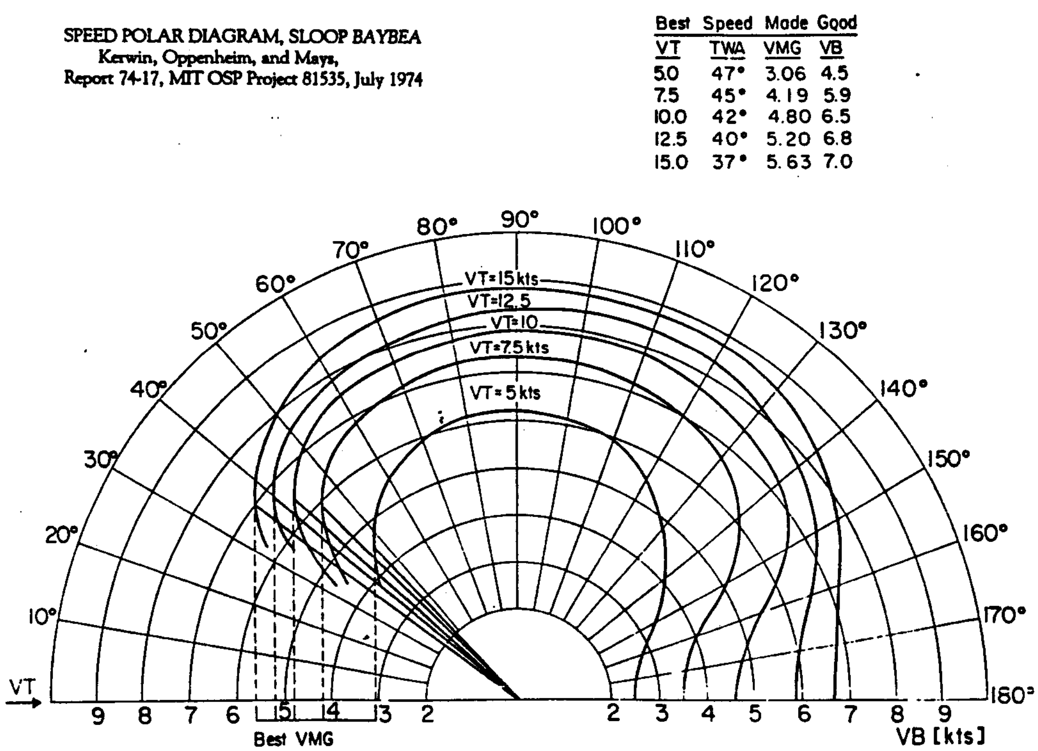


Figure 19. Sailplan for best VMG is 24568 upwind, 178 downwind



(Vt)60 kts	gamma deg	VMG kts	Vs kts	beta deg
5.0	58.6	2.22	4.26	28.0
7.5	54.6	2.85	4.92	28.0
10.0	53.1	3.64	6.07	28.0
15.0	49.3	5.12	7.84	28.0

Figure 20. Schooner *Brilliant* speed polar diagram.



VT	TWA	VMG	VB
5.0	47°	3.06	4.5
7.5	45°	4.19	5.9
10.0	42°	4.80	6.5
12.5	40°	5.20	6.8
15.0	37°	5.63	7.0

Figure 21. Sloop *Baybea* speed polar diagram.

CONCLUDING REMARKS

1. Smooth-water sail coefficients, a VPP , and a speed polar diagram were determined for schooner *Brilliant*.

2. The VPP did a reasonable job of predicting ship speed at the 46 actual sailing test points, with standard deviation of .028 (after eliminating three outlying points taken early in the sailing program).

3. The maximum predicted upwind ship speed and VMG to windward are attained with the fisherman and four lowers.

4. The maximum speed on reaches and maximum VMG downwind are achieved with the big jib, golliwobbler, and mainsail..

5. Sail plans 1568 and 24578 seem to provide little or no speed advantage over the ranges tested. There is uncertainty in this conclusion due to the small number of sailing runs performed with these sail plans; additional tests would be of interest. The big jib (sail code 1) used in the sailing tests was cut quite full. A flatter genoa jib cut for windward work might make a sail plan such as 1568 best to windward in light winds.

6. Righting moment is for all practical purposes independent of ship speed for *Brilliant*.

7. Leeway angle is approximately proportional to (heel)/(speed $^{1.8}$) for *Brilliant*. Leeway seldom exceeds 4^{o} except at lowest ship speeds.

8. The depth to center of hull lateral resistance (LRD) is about 32% of maximum draft.

9. Heel reduces *Brilliant* resistance force slightly. Rudder angle increases it slightly.

10. Comparison of schooner *Brilliant* and sloop *Baybea* speed polar diagrams indicate the following:

(a) Sailing upwind, the schooner and sloop both point higher as wind speed increases, for best velocity made good to windward. The sloop outpoints the schooner at all wind speeds, by about 10 degrees. The schooner is definitely at a disadvantage upwind, as tested.

(b) With the 24568 sail plan at 15 knots true wind speed, the schooner speed declines less rapidly than the sloop speed as the true wind angle increases beyond 80 degrees. The schooner is at its best versus the sloop when reaching, particularly at high wind speeds.

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Mark Starr, MSM, lines programs.

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APPENDIX 1: RESISTANCE CALCULATIONS

Total hull resistance can be expressed in the form of a dimensionless coefficient, C_t, that is the sum of contributing coefficients of resistance.

$$C_t = C_u + C_a + C_r \quad \text{(A1)}$$

$C_u = R_u/(\rho_w U_s^2 S/2)$ upright resistance

$C_a = R_a/(\rho_w U_s^2 S/2)$ attitude (λ and ϕ)

$C_r = R_r/(\rho_w U_s^2 S/2)$ rudder angle

The upright resistance coefficient can be further separated into two contributing coefficients:

$$C_u = C_v + C_w \quad \text{(A2)}$$

$C_v = R_v/(\rho_w U_s^2 S/2)$ friction

$C_w = R_w/(\rho_w U_s^2 S/2)$ upright wave-making

C_v is presumed to scale with N_{Re}, while C_w, C_a, and C_r scale with Froude Number, N_{Fr}.

Calculation Of C_v, C_w, and C_u

C_v for model and ship is:

$C_{v,m} = C_f(1+k) + C_{stm}$ model, at model N_{Re} (A3)

$C_{v,s} = C_f(1+k)$ ship, at ship N_{Re} (A4)

C_{stm} is the friction coefficient due to the turbulence stimulators and C_f is the flat plate drag coefficient given by the Schoenherr equation (e.g., Goldstein, 1950),

$$0.242/\mathrm{Sqrt}(C_f) = \mathrm{Log10}(N_{Re} C_f) \quad \text{(A5)}$$

The form factor, k, accounts for the difference between frictional resistance for a flat plate and the actual hull form of the ship of the same wetted area.

At very low speeds the wave-making resistance is assumed zero. k can therefore be determined by examining the resistance data at very low speeds.

The value for C_{stm} in the *Brilliant* tank tests is 0.00030. The value for k is 0.168, based on the behavior of upright resistance at low model speeds (Figure 6). k evaluated using the Prohaska technique (Teeters, CSYS 1993) was nearly the same.

C_w was found to fit a 7th-order polynomial function of Froude Number, based on the tank data.

$$C_w = \sum_{j=1}^{j=7} a_j N_{Fr}^{4j} \quad \text{(A6)}$$

$a_1 = 0.0901217$	$a_5 = -1.0365*10^7$
$a_2 = 23.8328$	$a_6 = 1.02037*10^8$
$a_3 = -4780.43$	$a_7 = 1.22825*10^7$
$a_4 = 360437$	

Combining these results, Figure 6 shows the curve

$$C_u = 1.168\, C_f + C_w \quad \text{(A7)}$$

$C_{u,s}$ is then calculated from Equation (A7), where C_f is evaluated, using Equation (A5), at the ship N_{Re}, and C_w is evaluated, using Equation (A6), at the ship N_{Fr}.

Calculation of $C_{a,s}$

Examination of tank data at four ship speeds, versus leeway angle and heel angle, resulted in empirical equation (A8) for the effect of attitude:

For N_{Fr} = 0.16 to 0.42 (V_s = 3.8 to 10 knots):

$$C_{a,s}/C_{u,s} = L_1 \lambda^{2.1} + P_2 \phi \quad \text{(A8)}$$

$$L_1 = 0.0350 - 0.0608\, N_{Fr}$$

$$P_2 = -.00116\, \lambda$$

The values fit the tank data, with standard deviation of 0.044 in the ratio of predicted to actual $C_{a,s} / C_{u,s}$.

Calculation of $C_{r,s}$

The effect of rudder angle is 1% to 8% increase in ship resistance at 4 degrees rudder, depending upon ship speed (largest at lowest speed). These influences are incorporated in empirical equation (A9):

For N_{Fr} 0.16 to 0.42 (V_s 3.8 to 10 knots):

$$C_{r,s}/(C_{a,s}+C_{u,s}) = [e^{-(N_{Fr})/0.132}]\,[(D/8)^{1.35}] \quad \text{(A9)}$$

The ship resistance increase, due to rudder angle, did not depend significantly upon heel and leeway.

APPENDIX 2: SIDE-FORCE COEFFICIENT C_H

The balance of transverse moments for the model is

$$YW(\mathrm{Cos}\,\phi) + (a+LRD_m)(\mathrm{Sec}\,\phi)F_m = g_m(\mathrm{Sin}\,\phi)\Delta_m + b_m(\mathrm{Sin}\,\phi)\Delta_m \quad \text{(A10)}$$

The dynamometer is attached at the level of the DWL, so that the dimension "a" is zero.

The balance of transverse moments for the ship is

$$(CEH)(\text{Sec}\,\phi)F_S + (LRD)_S(\text{Sec}\,\phi)F_S = g_S(\text{Sin}\,\phi)\Delta_S + b_S(\text{Sin}\,\phi)\Delta_S \qquad (A11)$$

Moment arms LRD, g_m, g_s, b_s and b_m are arbitrarily measured from the DWL.

The left side of (A10) and (A11) is the heeling moment, and the right side is the righting moment due to hull weight acting downward at the hull center of gravity and upward at the center of buoyancy.

Calculation of Sailing Equilibrium F_q and λ_q

The first condition considered is zero rudder and fixed CEH, of 27.75 ft. (four lowers sail plan). This condition is denoted by subscript $(\ )_{ref}$.

The model speed is identified here by the ship speed, V_S, at same N_{Fr}, three times model speed.

Expand the measured H_m, side force relative to path, and R_m, resistance force, to obtain H_e and R_e. Then calculate F_e, for each tow tank run at known λ, ϕ, V_S:

$$F_e = H_e \text{Cos}\,\lambda + R_e \text{Sin}\,\lambda. \qquad (A12)$$

Next write Equation (A10), the moment balance for the tank model in expanded form, (A10a).

$$[YW(\text{Cos}\,\phi)]_e + (LRD)_e(\text{Sec}\,\phi)F_e = g_e(\text{Sin}\,\phi)\Delta_e + b_e(\text{Sin}\,\phi)\Delta_\varepsilon \qquad (A10a)$$

and compare it with Equation (A11), the moment balance for the ship.

At any fixed ship speed and fixed heel there is one value of transverse moment for which side force F_e due to hull characteristics is equal to side force F_S due to aerodynamics of the rig. Consider this condition, and note also that $(LRD)_e = (LRD)_S$, $b_e = b_S$, and $\Delta_e = \Delta_S$. Subtract (A11) from (A10a) to eliminate LRD and rearrange to group the remaining expanded tank moments (hull) on the left and ship moments on right:

$$[YW(\text{Cos}\,\phi)]_e - g_e(\text{Sin}\,\phi)\Delta_e = (CEH)(\text{Sec}\,\phi)F_S - g_S(\text{Sin}\,\phi)\Delta_S \qquad (A13)$$

Call the left side of (A13) moment M_e, the right M_S:

$$M_e = [YW(\text{Cos}\,\phi)]_e - g_e(\text{Sin}\,\phi)\Delta_e \qquad (A14)$$
$$M_S = (CEH)(\text{Sec}\,f)F_S - g_S(\text{Sin}\,\phi)\Delta_S \qquad (A15)$$

The second term on the right in each equation is now seen to be equivalent to placing the c.g. of both model and ship at the same scaled location, in this case at the DWL. For sailing equilibrium, M_e must equal M_s.

To visualize equilibrium, see Figure 9, the F-M diagram. The sailing equilibrium line is drawn to pass through the intersections of the rig characteristic F-M lines and the hull characteristic F-M lines.

The value of sailing equilibrium leeeway, $\lambda_{q,ref}$, at each ϕ and V_s, is determined by interpolating along the hull characteristic lines.

By repeating this process for several ship speeds the data on sailing equilibrium is reduced to several tables. Then algorithms are fitted to these tables, in order to permit calculation of $\lambda_{q,ref}$, $F_{q,ref}$, and $H_{q,ref}$ for each sailing test, where ϕ and V_S are known. For $\lambda_{q,ref}$:

$$\lambda_{q,ref} = f - 0.038\,f^2 - 0.00028\,f^3 \qquad (A16)$$

where $\quad f = 3.8\,(\phi^{c}) / (N_{Fr}/.04191)^{1.8} \qquad (A17)$

and $\quad c = 1.6\,(1 - .037\,[|(N_{Fr}/.04191) - 5|^{1.41}]) \qquad (A18)$

The curve fits for $F_{q,ref}$ and $H_{q,ref}$ are

$$F_{q,ref} = (350\text{Cos}\,\phi - 175)(\phi) \qquad (A19)$$
$$H_{q,ref} = (360\text{Cos}\,\phi - 187)(\phi) \qquad (A20)$$

Note that $F_{q,ref}$ and $H_{q,ref}$ are independent of ship speed, for all practical purposes.

Final C_H, With Effects Of Rudder Angle and CEH

In Figure 9, the F-M diagram, note the following effects:

For an increase in CEH, F_q moves down along a line of hull ϕ=constant. F_q decreases and λ_q decreases.

For an increase in rudder angle D, F_e moves up approximateky along a line of hull ϕ = constant.
- F_e increases and M_e decreases slightly.
- Lines of tank λ=constant move upward.
- Sailing equilibrium side force F_q does not change, but now corresponds to lower λ_q at each ϕ.

To quantify these effects, first consider the change in leeway angle due to a small change in side force in the tank at ϕ=constant, V_s=constant, and D=0. The change can be described by a parameter X, defined as

$$X(\lambda, V_s, \phi) = [(\delta\lambda/\lambda)/(\delta F/F)]_{\phi=const,\ Vs=const,\ D=0} \quad (A21)$$

The following fit to the data was found:

If $\lambda<3.355$, any N_{Fr}, $X=0.119 + 0.146\ \lambda$ (A22)
If $\lambda>3.355$, $N_{Fr}<=.2934$, $X=0.609+0.0552(\lambda - 3.355)$

If $\lambda>3.355$, $N_{Fr}>.2934$, $X=0.609+ (0.0356+[[0.0552((N_{Fr}/.04191) -7)])(\lambda-3.355)$

For rudder angle increase alone, at constant ϕ, both F_e and λ change. F_e changes by $F_{e,D2}-F_{e,D1}$. The fit is

$$F_{e,D2}-F_{e,D1} = 31D^{1.22} + 0.613D^{1.447}\ \phi \quad (A23)$$

The change in λ_q due to rudder angle change is then

$$\lambda_{q,D2} - \lambda_{q,D1} = -(X)(\lambda_{q,D1})(F_{e,D2}-F_{e,D1})/F_{e,D1} \quad (A24)$$

Next consider the change in λ_q due to a small change in CEH. Equation (A15) shows that, to first order, for constant ϕ, and using the approximation M=constant,

$$(F_{q,CEH2}-F_{q,CEH1})/F_{q,CEH1} = -(CEH2-CEH1)/(CEH1) \quad (A25)$$

The change in λ_q due to CEH change is then

$$\lambda_{q,CEH2}-\lambda_{q,CEH1} = -(X)(\lambda_{q,CEH1})(CEH2-CEH1)/(CEH1) \quad (A26)$$

The final value of H_q is obtained by inserting the final F_q, final λ_q, and final R_q values in (A27):

$$H_q = (F_q - R_q \sin\lambda_q)/(\cos\lambda_q) \quad (A27)$$
where $R_q = (\rho_w U_s^2 S/2)(C_{t,q})$

The sail side-force coefficient C_H can now be calculated by inserting H_q from (A27) into (3).

APPENDIX 3: DEPTH TO CENTER OF LATERAL RESISTANCE

The transverse moment balance, from (A10), is

$$YW(\cos\phi) + (LRD)_m(\sec\phi)F_m = g_m(\sin\phi)D_m + b_m(\sin\phi)D_m \quad (A28)$$

There are two unknowns, $(LRD)_m$ and b_m. If the shape of the water wave at the hull surface were known, then b_m could be calculated from a computer program such as Nautilus. (A28) could then be solved for LRD. So far, attempts to determine the wave shape from photos taken during the testing were not satisfactory. An alternative approach is described below.

Combining Equations (A14) and (A10a),

$$M_e + (LRD)_e(\sec\phi)F_e = b_e(\sin\phi)D_e \quad (A29)$$

Since b_e and $(LRD)_e$ can be assumed to be near constant for constant ship speed, heel, and rudder, then an expression for $(LRD)_e$ can be determined by differentiating (A32).

$$\delta F_e/\delta M_e = -1/[(LRD)_e \sec\phi] \quad (A30)$$

But in the F-M diagram, Figure 9, a straight line of the form $F_e = d + nM_e$ describes the variation of F_e with M_e, at constant V_s, ϕ, and rudder. Then $\delta F_e/\delta M_e = n$ and (A33) becomes

$$(LRD)_e = -(\cos\phi)/n \quad (A31)$$

An empirical expression for calculation of LRD for Brilliant, based on the values of n found for several heel angles at 5 knots, 7 knots, and 9 knots, is:

$$LRD = 5 - 0.32V_s + 0.025\ \phi \quad (A32)$$